

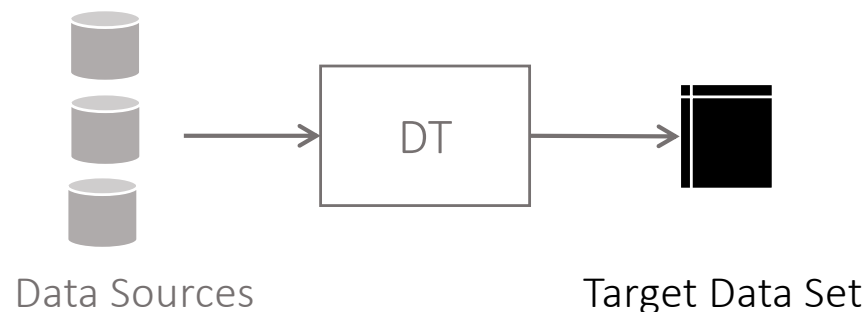
# DISTRIBUTION TAILORING

# MOTIVATION

- Distribution requirements on data sets
  - reducing model error (for feature slices)
  - showing adequate consideration of minority groups
- Sources of data
  - explicitly collected by the data scientist
  - secondary data, collected for some other purpose
- Can data from multiple sources be put together to build a data set with a desired distribution?
  - Data Distribution Tailoring (DT)

# QUERY MODEL

- User's query: target schema and distribution requirements
- Target schema contains some sensitive attributes that identify the groups.
- A distribution requirement specified over some groups
  - Count requirements: group ratio + target size



Schema: movie\_title, actor\_name,  
gender, race, ...

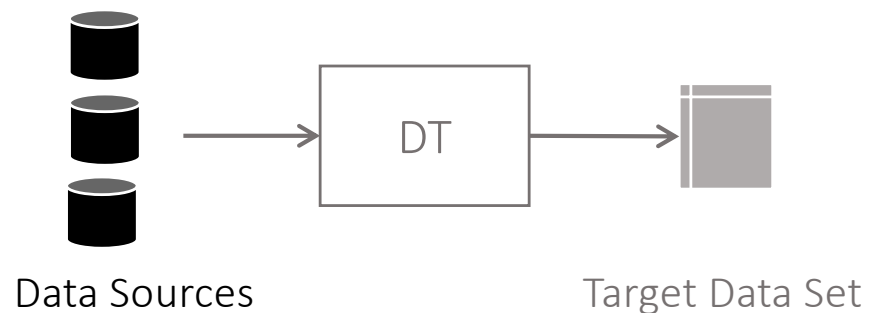
Distbn Requirements:  
WM: 1K, NWF: 1K, ...

# DATA MODEL

- A collection of data sources
- Each source has the same schema as the user's query schema.
  - Each tuple of a source can be associated with a group.
- We assume a tuple-at-a-time access to a source.

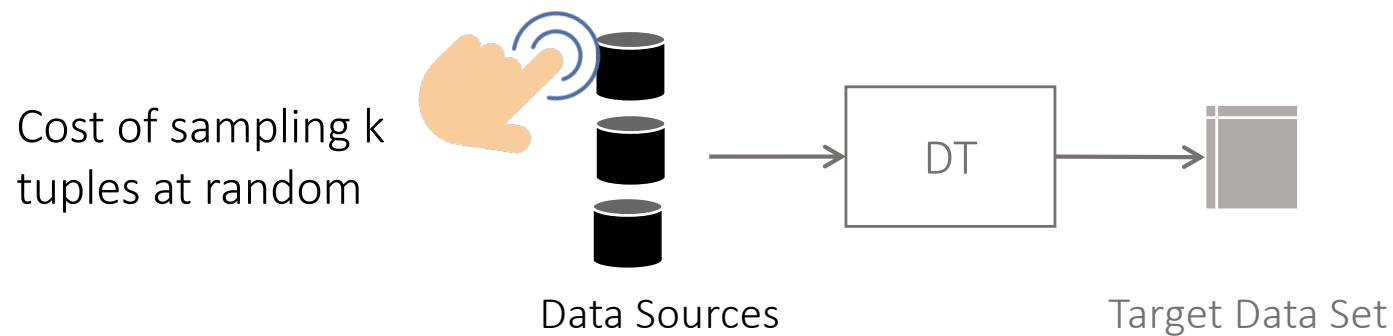
Schema: movie\_title, actor\_name,  
gender, race, ...

SPJ views over data lakes, web  
services, data markets, or data  
brokers



# COST MODEL

- Obtaining samples from different data sources is not for free.
- Samples are associated with a cost: monetary, computation, memory or network access.



# DATA DISTRIBUTION TAILORING (DT)

- Given sources  $L = \{D_1, \dots, D_n\}$  with their costs  $\{C_1, \dots, C_n\}$ , and count requirements  $\{Q_1, \dots, Q_m\}$  on groups  $\{G_1, \dots, G_m\}$ , our goal is to query different sources in  $L$ , in a sequential manner, in order to collect samples that fulfill the count requirement, while the expected total query cost is minimized.

# DT ALGORITHM

Input: data sources  $L=\{D_1, \dots, D_n\}$  and  $\{C_1, \dots, C_n\}$   
counts  $\{Q_1, \dots, Q_m\}$  over  $\{G_1, \dots, G_m\}$ ;

Output:  $O$ , the target data set

1:  $O \leftarrow \{\}$ ,  $\text{cost} \leftarrow 0$

2: while( $Q_j > 0$ ) do

3:      $D_i, C_i \leftarrow \text{select\_optimal\_source}()$

4:      $s \leftarrow \text{Query}(D)$

5:      $j \leftarrow \text{Group}(s)$

6:     if( $s \notin O$  AND  $Q_j > 0$ ) then

7:         add  $s$  to  $O$ ;

8:          $Q_j \leftarrow Q_j - 1$

9:      $\text{cost} \leftarrow \text{cost} + C_i$

10: return  $O$

# VERSIONS OF DT

- **Known** source distributions
- **Unknown** source distributions



# DT: KNOWN DISTRIBUTIONS

- Notations
  - $Q = \{Q_1, \dots, Q_m\}$ : count requirements on  $m$  groups
  - $C_i$ : cost of  $D_i$
  - $P_i^j$ : prob of collecting  $G_j$  from  $D_i$ 
    - $N_i$ : #tuples in data source  $D_i$
    - $N_i^j$ : #tuples in  $D_i$  that belong to  $G_j$
  - $F(Q)$ : min expected cost of a target with counts  $Q$
- How to compute  $F(Q)$ ?
  - Think recursively. Consider the probability of obtaining a fresh and useful tuple. Include the case when a tuple is not useful.

# KNOWN DT: COST FUNCTION

$F(Q)$ : min expected cost of a target with counts  $Q$

$F_j(Q) = F(Q_1, \dots, Q_{j-1}, \dots, Q_m)$

- Take a sample from  $D_i$

$$C_i + \sum_{j=1, Q_j > 0}^m P_i^j F_j(Q) + (1 - \sum_{j=1, Q_j > 0}^m P_i^j) F(Q)$$

Cost of sample
prob. of a seen or useless sample of  $Q_c$

Exp. cost of the rest of data collection if a fresh sample is obtained  $G_j$  from  $D_i$ 
Exp. cost of the rest of data collection if sample is not fresh or does not help with target

Expected remaining cost

# KNOWN DT: COST FUNCTION

- Source selection strategy

$$\min_{\forall D_i} (C_i + \sum_{j=1, Q_j > 0}^m P_i^j F_j(Q) + (1 - \sum_{j=1, Q_j > 0}^m P_i^j) F(Q))$$

- What kind of assumption do we make here on  $P_i^j$ ?
- Which algorithmic technique can we use to solve this optimization problem?

# A DYNAMIC PROGRAMMING SOLUTION

	cost	groups	
sources	$C_i$	$G_1$	$G_2$
$D_1$	2	0.2	0.8
$D_2$	3	0.4	0.6

cost of obtaining a tuple of  $G_1$  from  $D_1$ :  $2/0.2=10$   
 cost of obtaining a tuple of  $G_1$  from  $D_2$ :  $3/0.4=7.5$

$$F(1,0) = \min(2/0.2, 3/0.4) = 7.5 \leftarrow D_2$$

$$F(0,1) = \min(2/0.8, 3/0.6) = 2.5 \leftarrow D_1$$

Query:  $G_1: 1$  and  $G_2: 1$

$F(1,1)$ : the cost of a target with  $G_1: 1$  and  $G_2: 1$

	$G_2$	
$G_1$		
	0	1
0	$F(0,0)=0$	$F(0,1)$
1	$F(1,0)$	$F(1,1) \checkmark$

*Note: Red arrows point from  $F(1,0)$  to  $F(0,1)$  (labeled  $D_2$ ) and from  $F(1,0)$  to  $F(1,1)$  (labeled  $D_1$ ).*

select  $D_1$ :  $2 + 0.2 F(0,1) + 0.8 F(1,0)$   
 select  $D_2$ :  $3 + 0.4 F(0,1) + 0.6 F(1,0)$

$$F(1,1) = \min(2 + 0.2 F(0,1) + 0.8 F(1,0), 3 + 0.4 F(0,1) + 0.6 F(1,0)) = 8.4 \leftarrow D_1$$

# DP COMPLEXITY

- What is the complexity of this DP algorithm?

# DP COMPLEXITY

- Pseudo-polynomial time complexity

$$O(n m \prod_{i=1}^m Q_i)$$

- Not practical for realistic settings

# EQUI-COST BINARY DT

- Let's consider a common and simple setting
- Groups  $\{G_1, G_2\}$  with counts  $\{Q_1, Q_2\}$  and all source costs are equal.
- $P_i^j$  : prob of collecting  $G_j$  from  $D_i$
- What is the cost of getting a fresh tuple of group  $G_j$  from  $D_i$ ?
- What is the best source for group  $G_j$ ?

# EQUI-COST BINARY DT

- Groups  $\{G_1, G_2\}$  with counts  $\{Q_1, Q_2\}$  and all source costs are equal.
- Cost of getting a fresh tuple of  $G_j$  from  $D_i$  (geometric distribution):
  - $\frac{N_i}{N_i^j - O_i^j}, O_i^j$ : #seen tuples of  $G_j$  from  $D_i$
- The best source for  $G_j$ :  $D_{*j} = \underset{\forall D_i}{\operatorname{argmax}} \left( \frac{N_i^j - O_i^j}{N_i} \right)$



# EQUI-COST BINARY DT

- Groups  $\{G_1, G_2\}$  with counts  $\{Q_1, Q_2\}$  and all source costs are equal.
- Cost of getting a fresh tuple of  $G_j$  from  $D_i$  (geometric distribution):
  - $\frac{N_i}{N_i - O_i^j}, O_i^j$ : #seen tuples of  $G_j$  from  $D_i$
- The best source for  $G_j$ :  $D_{*j} = \underset{\forall D_i}{\operatorname{argmax}} \left( \frac{N_i - O_i^j}{N_i} \right)$

# OPTIMAL EQUI-COST BINARY

- Which source we should pick in each iteration?

# OPTIMAL EQUI-COST BINARY

- Hint: we can find the best source for each group:  $D_{*1}$  and  $D_{*2}$

$$D_{*1} = D_i \text{ and } P_{*1} = \frac{N_i^1 - O_i^1}{N_i}$$

$$D_{*2} = D_j \text{ and } P_{*2} = \frac{N_j^2 - O_j^2}{N_j}$$

- Which incurs lower cost?

# OPTIMAL EQUI-COST BINARY

- Find the best source for each group:  $D_{*1}$  and  $D_{*2}$

$$D_{*1} = D_i \text{ and } P_{*1} = \frac{N_i^1 - O_i^1}{N_i}$$

$$D_{*2} = D_j \text{ and } P_{*2} = \frac{N_j^2 - O_j^2}{N_j}$$

**Theorem.** Consider the DT problem under the availability of group distributions where there are two groups and the costs for querying data sources are equal. Let  $G_1$  be the minority, i.e.  $P_{*1} \leq P_{*2}$ . Selecting  $D_{*1}$  to query at current iteration is optimal.

# DT FOR OTHER SETTINGS

- General DT: non-binary case ( $m > 2$ ) with unequal source costs
  - approximation algorithm with cost upper bound analysis
- Unknown DT
  - An exploration-exploitation solution based on the Multi-Arm Bandit framework

# OPTIMAL EQUI-COST BINARY

- Proof by contradiction
- Intuition: piggy-backing
  - while sampling from the minority group, we collect items of the majority group.

# PROOF SKETCH

- Proof by contradiction
- Let  $D_{*1} = D_i$ . Suppose  $A_1$  that select  $D_i$  is not optimal. Suppose the optimal algorithm  $A_2$  selects  $D_{i \neq j}$ . We show that the expected cost of  $A_1$  cannot be less than  $A_2$ . Let  $P' = \frac{N_j^1 - O_j^1}{N_j}$ . Note  $P' \leq P_{*1}$ .

$$F_i(Q_1, Q_2) = P_{*1} F(Q_1-1, Q_2) + (1-P_{*1}) F(Q_1, Q_2-1)$$

$$F_j(Q_1, Q_2) = P' F(Q_1-1, Q_2) + (1-P') F(Q_1, Q_2-1)$$

$$\begin{aligned} B &= F_j(Q_1, Q_2) - F_i(Q_1, Q_2) \\ &= (P_{*1} - P')(F(Q_1, Q_2-1) - F(Q_1-1, Q_2)) \end{aligned}$$

# PROOF SKETCH

$$F(Q_1-1, Q_2) = F(Q_1-1, Q_2-1) + F(0,1)$$

$$F(Q_1, Q_2-1) = F(Q_1-1, Q_2-1) + F(1,0)$$

- Since G1 is the minority,  $F(0, 1) \leq F(1, 0)$ . Therefore  $B \geq 0$
- Since the expected cost of  $A_1$  cannot be less than that of  $A_2$ , selecting  $D_i = D_{*1}$  to query at iteration  $i$  is an optimal solution.



# EQUI-COST BINARY DT ALGORITHM

**Input:** number of items from  $Q = \{Q_1, Q_2\}$ ;

data sources  $L = \{D_1, \dots, D_n\}$

**Output:**  $O$ , the target data set

1:  $O \leftarrow \{\}$

2: **while**( $Q_1 > 0$  AND  $Q_2 > 0$ ) **do**

3:      **$D \leftarrow$  source with max ratio of undiscovered  $G_1$**

4:      **$D' \leftarrow$  source with max ratio of undiscovered  $G_2$**

5:      **$D'' \leftarrow$  source ( $D$  or  $D'$ ) with the minority group**

6:      $s \leftarrow \text{Query}(D'')$

...

# GENERAL NON-BINARY DT

- Multiple groups  $\{G_1, \dots, G_m\}$  with count requirements  $\{Q_1, \dots, Q_m\}$  and source costs are not equal.
- Brainstorming for an algorithm for the general non-binary DT.

# GENERAL NON-BINARY DT

- Multiple groups  $\{G_1, \dots, G_m\}$  with count requirements  $\{Q_1, \dots, Q_m\}$  and source costs are not equal.
- For group  $G_j$ , what is the most cost-effective data source?
- How can we use the cost-effective data sources to fulfill the count requirements?

# GENERAL NON-BINARY DT

- For group  $G_j$ , the most cost-effective data source is

$$D_{*j} = \operatorname{argmax}_{\forall D_i} \frac{N_i^j}{N_i \cdot C_i}$$

# GENERAL DT ALGORITHM

- Select the most cost-effective source for  $G_j$  (namely  $D_{*j}$ ) and commit to it.
- Query the data source  $D_{*j}$  for group  $G_j$ 
  - Maintain the tuples of other groups (*piggybacking*)
- Repeat until the target specified by the count description  $[Q_1, \dots, Q_m]$  is collected.

## PROJECT 2: VARIATIONS OF DT

- Given sources  $L = \{D_1, \dots, D_n\}$  with their costs  $\{C_1, \dots, C_n\}$ , and count requirements  $\{Q_1, \dots, Q_m\}$  on groups  $\{G_1, \dots, G_m\}$ , our goal is to query different sources in  $L$ , in a sequential manner, in order to collect samples that fulfill the count requirement, while the expected total query cost is minimized.
- Generalize the problem to
  - fixed  $> 1$  number of samples at each iteration
  - arbitrary number of samples at each iteration
  - count requirements on multiple groups (e.g. 100 of gender=F and 100 of gender=M as well as 100 of race=W and 100 of race=NW)
  - overlapping sources